CALCULUS AB
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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76. A particle moves along a straight line so that at time $t > 0$ the position of the particle is given by $s(t)$, the velocity is given by $v(t)$, and the acceleration is given by $a(t)$. Which of the following expressions gives the average velocity of the particle on the interval $[2, 8]$?

(A) $\frac{1}{6}\int_2^8 a(t)\,dt$

(B) $\frac{1}{6}\int_2^8 s(t)\,dt$

(C) $\frac{s(8)-s(2)}{6}$

(D) $\frac{v(8)-v(2)}{6}$

(E) $v(8)-v(2)$

Average value on $[a, b]$ for $f(x)$ is...

$$\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{F(b)-F(a)}{b-a}$$

Average velocity on $[2, 8]$ is

$$\frac{1}{8-2}\int_2^8 v(t)\,dt = \boxed{\frac{s(8)-s(2)}{8-2}}$$

Note: $\int v(t)\,dt \Rightarrow s(t) + c$

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77. If $\sin\left(\frac{1}{x^2+1}\right)$ is an antiderivative for $f(x)$, then $\int_1^2 f(x)\,dx =$

(A) -0.281 (B) -0.102 (C) 0.102 (D) 0.260 (E) 0.282

antiderivative for $f(x)$ is $\sin\left(\frac{1}{x^2+1}\right)$, therefore

$$\int_1^2 f(x)\,dx = \sin\left(\frac{1}{2^2+1}\right) - \sin\left(\frac{1}{1^2+1}\right)$$

$$\sin\left(\frac{1}{5}\right) - \sin\left(\frac{1}{2}\right)$$

$$\boxed{-0.281}$$

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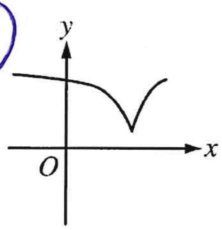
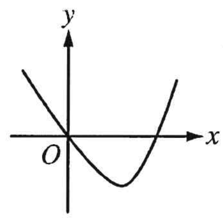
78. The function f is differentiable and increasing for all real numbers x, and the graph of f has exactly one point of inflection. Of the following, which could be the graph of f', the derivative of f?

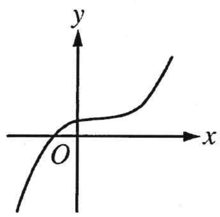
(A)



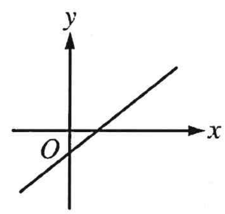
(B)



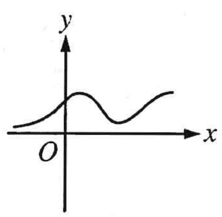
(C)



(D)



(E)



1) f is differentiable ⇒ The derivative exists: ABCDE

2) f is increasing ⇒ The derivative is positive: A+E only

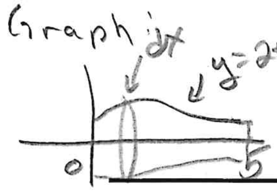
3) Exactly 1 point of inflection ⇒ 2^{nd} derivative changes sign exactly 1 time (+ to − or − to +). This means the 1^{st} derivative can only change from increasing to decreasing (or vice versa) only 1 time as well.

$\boxed{\text{A is only option}}$ because E changes multiple times ⇒ multiple inflection points.

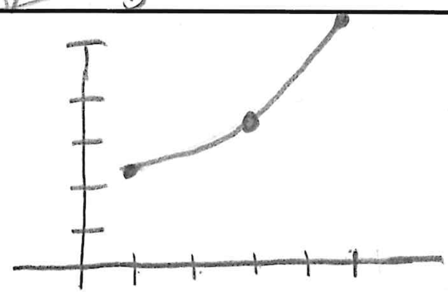
79. A vase has the shape obtained by revolving the curve $y = 2 + \sin x$ from $x = 0$ to $x = 5$ about the x-axis, where x and y are measured in inches. What is the volume, in cubic inches, of the vase?

(A) 10.716 (B) 25.501 (C) 33.666 (D) 71.113 (E) 80.115

Graph it



$$\pi \int_0^5 (2+\sin x)^2 dx = \boxed{80.115}$$



x	$f(x)$
1	2.4
3	3.6
5	5.4

Look up Mean Value Theorem

80. The table above gives selected values of a function f. The function is twice differentiable with $f''(x) > 0$. Which of the following could be the value of $f'(3)$?

(A) 0.6 (B) 0.7 (C) 0.9 (D) 1.2 (E) 1.5

Concave up

Tough question here → follow closely...

› According to the mean value theorem, there exists a point between $x = 1$ and $x = 3$ whose slope (derivative) is $m = .6$. This is because $\frac{f(3) - f(1)}{3-1} = \frac{3.6 - 2.4}{2} = \frac{1.2}{2} = .6$

> Similarly, the MVT guarantees a point between 3 and 5 to have a slope (derivative) of $m = .9$.

> Since $f''(x) > 0$, the derivative $f'(x)$ is always increasing. $m = .6$ before $x = 3$ and $m = .9$ after $x = 3$, the only option for m @ $x = 3$ would be $f'(3)$ could be $\boxed{.7}$.

Told you this was a tough question.

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81. At time $t = 0$ years, a forest preserve has a population of 1500 deer. If the rate of growth of the population is modeled by $R(t) = 2000e^{0.23t}$ deer per year, what is the population at time $t = 3$?

(A) 3987 (B) 5487 (C) 8641 (D) 10,141 (E) 12,628

R(t) is the rate → derivative → velocity

∫ velocity ⇒ displacement

$$1500 + \int_0^3 R(t)\,dt = \boxed{10{,}141.004}$$

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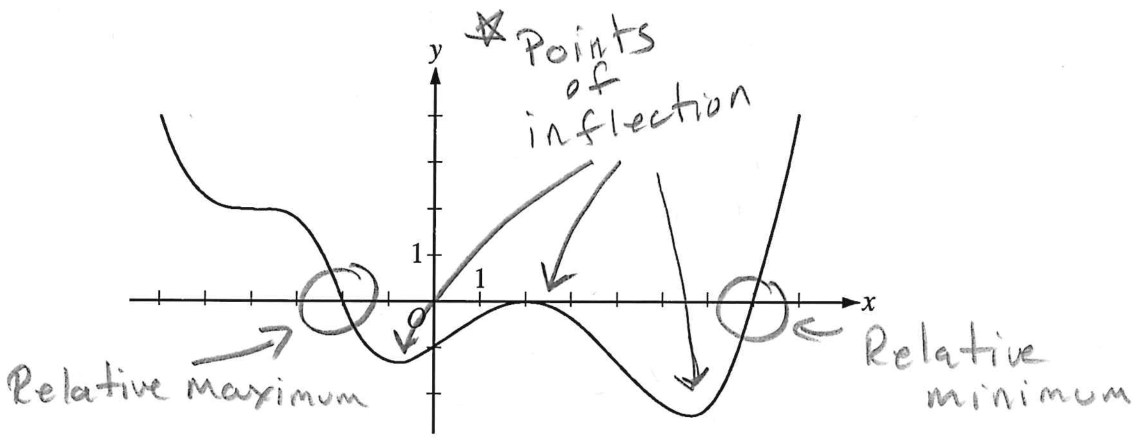

Graph of f'

82. The figure above shows the graph of f', the derivative of function f, for $-6 < x < 8$. Of the following, which best describes the graph of f on the same interval?

(A) 1 relative minimum, 1 relative maximum, and 3 points of inflection

(B) 1 relative minimum, 1 relative maximum, and 4 points of inflection

(C) 2 relative minima, 1 relative maximum, and 2 points of inflection

(D) 2 relative minima, 1 relative maximum, and 4 points of inflection

(E) 2 relative minima, 2 relative maxima, and 3 points of inflection

* The derivative changing from increasing to decreasing or from decreasing to increasing implies the 2nd derivative is changing from positive to negative or − to +.

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83. Let f and g be continuous functions such that $\int_0^6 f(x)\,dx = 9$, $\int_3^6 f(x)\,dx = 5$, and $\int_3^0 g(x)\,dx = -7$. What is the value of $\int_0^3 \left(\frac{1}{2}f(x) - 3g(x)\right) dx$?

(A) -23 (B) -19 (C) $-\frac{17}{2}$ (D) 19 (E) 23

Since $\int_0^6 f(x)dx = 9$ and $\int_3^6 f(x)dx = 5$ then

$$\int_0^6 f(x)dx - \int_3^6 f(x)dx = 9 - 5 = 4$$

Therefore $\int_0^3 f(x)dx = 4$

bounds switched

Also $\int_3^0 g(x)dx = -7 \Rightarrow \int_0^3 g(x)dx = 7$

$$\int_0^3 \left(\tfrac{1}{2}f(x) - 3g(x)\right)dx = \tfrac{1}{2}\int_0^3 f(x)\,dx - 3\int_0^3 g(x)\,dx$$

$$= \tfrac{1}{2}(4) - 3(7)$$

$$= 2 - 21$$

$$= \boxed{-19}$$

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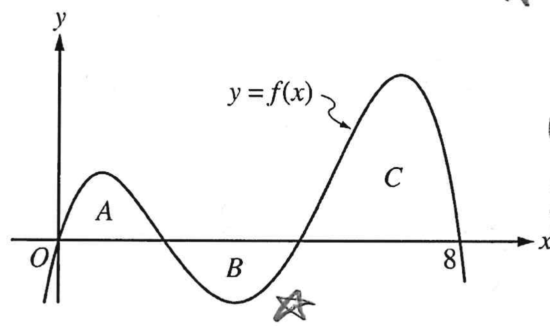

★ B → The area is 16, but since it's under the x-axis, its integral on that interval is $\underline{-16}$.

84. The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. The area of region A is 14, the area of region B is 16, and the area of region C is 50. What is the average value of f on the interval $[0, 8]$?

(A) 6 (B) 10 (C) $\frac{40}{3}$ (D) $\frac{80}{3}$ (E) 48

Avg value of f on $[0,8]$ $\Rightarrow \frac{1}{8-0}\int_0^8 f(x)\,dx = \frac{1}{8}(14 + -16 + 50)$

A B C

$= \frac{1}{8}(48) = \boxed{6}$

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85. A particle moves along the x-axis so that its velocity at time $t \geq 0$ is given by $v(t) = \frac{t^2 - 1}{t^2 + 1}$. What is the total distance traveled by the particle from $t = 0$ to $t = 2$?

(A) 0.214 (B) 0.320 (C) 0.600 (D) 0.927 (E) 1.600

total distance traveled $\Rightarrow \int_a^b |v(t)|\,dt$

$\int_0^2 \left|\frac{t^2-1}{t^2+1}\right| dt$ on calculator

$\boxed{.92729}$

$dy/dx = e^x$ m @ $x = k$ is e^k

86. Line ℓ is tangent to the graph of $y = e^x$ at the point (k, e^k). What is the positive value of k for which the y-intercept of ℓ is $\frac{1}{2}$?

point & slope

(A) 0.405
(B) 0.768
(C) 1.500
(D) 1.560
(E) There is no such value of k.

$y - y_1 = m(x - x_1)$ $y - e^k = e^k(x - k)$

$y - e^k = e^k x - e^k \cdot k$

$y = e^k x \boxed{- e^k k + e^k}$

Think... $y = mx + b$

$-e^k k + e^k$ is the "b" (y-intercept)

$-e^k k + e^k = \frac{1}{2}$ → Solve by graphing: $-e^k k + e^k - \frac{1}{2} = 0$

$\boxed{k = .76803905}$

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87. A differentiable function f has the property that $f'(x) \le 3$ for $1 \le x \le 8$ and $f(5) = 6$. Which of the following could be true?

I. $f(2) = 0$

II. $f(6) = -2$

III. $f(7) = 13$

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) II and III only

I. If $f(2) = 0$ and $f(5) = 6$, then the slope from $(2, 0)$ to $(5, 6)$ is $m = 2$. This agrees w/ $f'(x) \le 3$ on $[1, 8]$.

II. If $f(6) = -2$ and $f(5) = 6$, then the slope from $(6, -2)$ to $(5, 6)$ is $m = -8$. Again this agrees w/ $f'(x) \le 3$ on $[1, 8]$.

~~III~~. $f(7) = 13$ and $f(5) = 6 \Rightarrow m = \frac{7}{2}$

Since $7/2 > 3$, this is a contradiction to $f'(x) \le 3$.

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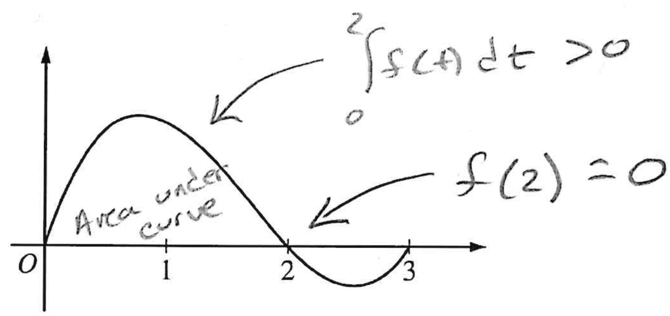


Graph of f

88. The graph of the differentiable function f is shown in the figure above. Let h be the function defined by $h(x) = \int_0^x f(t)\,dt$. Which of the following correctly orders $h(2)$, $h'(2)$, and $h''(2)$?

(A) $h(2) < h'(2) < h''(2)$

(B) $h'(2) < h(2) < h''(2)$

(C) $h'(2) < h''(2) < h(2)$

(D) $h''(2) < h(2) < h'(2)$

(E) $h''(2) < h'(2) < h(2)$

$h(2) = \int_0^2 f(t)\,dt > 0$

$h'(x) = f(x)$ by 2nd FTC.

Therefore $h'(2) = f(2) = 0$

$h''(x) = f'(x)$ (see above)

$h''(2) = f'(2)$ Since $f'(2)$ is the derivative of $f(t)$ at $t=2$, and $f(t)$ is decreasing at $t=2$, $f'(t)$ @ $t=2$ must be negative.

$h''(2) < 0$

$h''(2) < 0$, $h'(2) = 0$, $h(2) > 0$

Find where $y = e^x - 2$ intersects $y = \sin x \Rightarrow 1.0541271 = K$

89. What is the area of the region enclosed by the graphs of $y = e^x - 2$, $y = \sin x$, and $x = 0$?

(A) 0.239 (B) 0.506 (C) 0.745 (D) 2.340 (E) 3.472

$\int_0^K (\sin x - (e^x - 2))\, dx = \boxed{.744798}$

90. A particle moves along a line so that its velocity is given by $v(t) = -t^3 + 2t^2 + 2^{-t}$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

$a(t) = -3t^2 + 4t + 2^{-t}(\ln 2)(-1)$

derivative of $-t$

(A) $(0, 0.177)$ and $(1.256, \infty)$

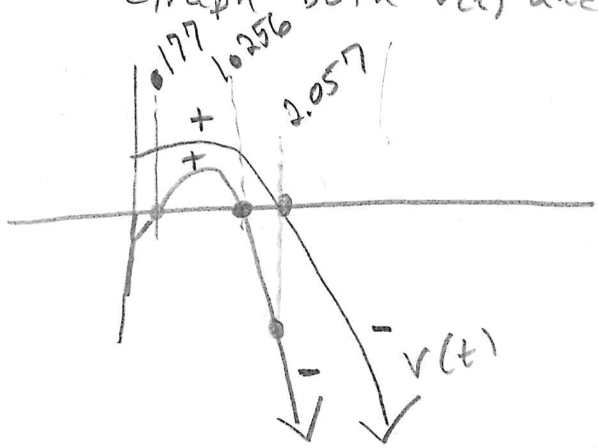
(B) $(0, 1.256)$ only

(C) $(0, 2.057)$ only

(D) $(0.177, 1.256)$ only

(E) $(0.177, 1.256)$ and $(2.057, \infty)$

Graph both $v(t)$ and $a(t)$



Speed is increasing when $v(t)$ + $a(t)$ have the same sign (both positive or both negative).

91. Let F be a function defined for all real numbers x such that $F'(x) > 0$ and $F''(x) > 0$. Which of the following could be a table of values for F ?

F(x) is increasing

B C D

(A)

x	$F(x)$
1	–3
2	–4
3	–6
4	–9

Decreasing

(B)

x	$F(x)$
1	–3
2	–1
3	3
4	19

+2
+4
+16

F(x) is concave up. More importantly, the derivative is increasing. This means the rate at which the function changes ($F'(x)$) must be increasing.
This is only occurring in option B.

(C)

x	$F(x)$
1	–3
2	0
3	3
4	6

+3
+3
+3

(D)

x	$F(x)$
1	–3
2	5
3	11
4	13

+2
+6
+2

(E)

x	$F(x)$
1	–3
2	–4
3	–3
4	0

Decreasing and Increasing

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x	$f(x)$	$g(x)$	$f'(x)$
−4	0	−9	5
−2	4	−7	4
0	6	−4	2
2	7	−3	1
4	10	−2	3

92. The table above gives values of the differentiable functions f and g, and f', the derivative of f, at selected values of x. If $g(x) = f^{-1}(x)$, what is the value of $g'(4)$?

(A) $-\frac{1}{3}$ (B $-\frac{1}{4}$ (C) $-\frac{3}{100}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

Remember the formula for the derivative of an inverse function...

$$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'(f^{-1}(x))}$$

$$\frac{d}{dx}\left[g(x)\right] = \frac{1}{f'(g(x))}$$ (use chart)

$$\frac{d}{dx}\left[g(4)\right] = \frac{1}{f'(g(4))} = \frac{1}{f'(-2)}$$

$$= \boxed{\frac{1}{4}}$$