CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. $\int_2^x \left(3t^2 - 1\right)dt =$

(A) $x^3 - x - 6$ (B) $x^3 - x$ (C) $3x^2 - 12$ (D) $3x^2 - 1$ (E) $6x - 12$

2. What is the slope of the line tangent to the graph of $y = \ln(2x)$ at the point where $x = 4$?

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) 4

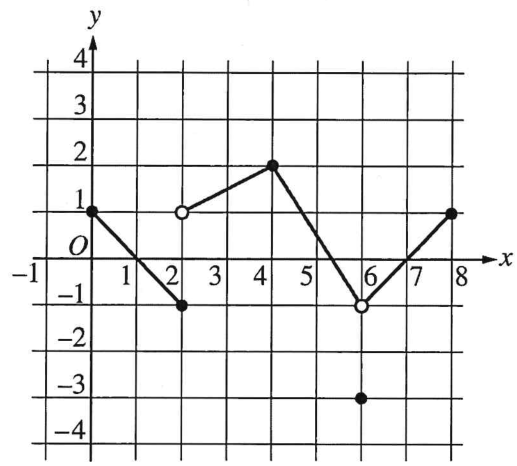
3. If $f(x) = 4x^{-2} + \frac{1}{4}x^2 + 4$, then $f'(2) =$

(A) -62 (B) -58 (C) -3 (D) 0 (E) 1

---

4. $\int_1^2 \frac{dx}{2x+1} =$

(A) $2\ln 2$ (B) $\frac{1}{2}\ln 2$ (C) $2(\ln 5 - \ln 3)$ (D) $\ln 5 - \ln 3$ (E) $\frac{1}{2}(\ln 5 - \ln 3)$



5. The figure above shows the graph of the function f. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x) = f(2)$

II. $\lim_{x \to 6^-} f(x) = \lim_{x \to 6^+} f(x)$

III. $\lim_{x \to 6} f(x) = f(6)$

(A) II only

(B) III only

(C) I and II only

(D) II and III only

(E) I, II, and III

GO ON TO THE NEXT PAGE.

6. $\frac{d}{dx}\left(\sin^3\left(x^2\right)\right) =$

(A) $\cos^3\left(x^2\right)$

(B) $3\sin^2\left(x^2\right)$

(C) $6x\sin^2\left(x^2\right)$

(D) $3\sin^2\left(x^2\right)\cos\left(x^2\right)$

(E) $6x\sin^2\left(x^2\right)\cos\left(x^2\right)$

GO ON TO THE NEXT PAGE.

7. $\lim_{x \to \infty} \frac{x^3}{e^{3x}}$ is

(A) 0 (B) $\frac{2}{9}$ (C) $\frac{2}{3}$ (D) 1 (E) infinite

---

8. Using the substitution $u = \sin(2x)$, $\int_{\pi/6}^{\pi/2} \sin^5(2x)\cos(2x)\, dx$ is equivalent to

(A) $-2\int_{1/2}^{1} u^5\, du$

(B) $\frac{1}{2}\int_{1/2}^{1} u^5\, du$

(C) $\frac{1}{2}\int_{0}^{\sqrt{3}/2} u^5\, du$

(D) $\frac{1}{2}\int_{\sqrt{3}/2}^{0} u^5\, du$

(E) $2\int_{\sqrt{3}/2}^{0} u^5\, du$

GO ON TO THE NEXT PAGE.

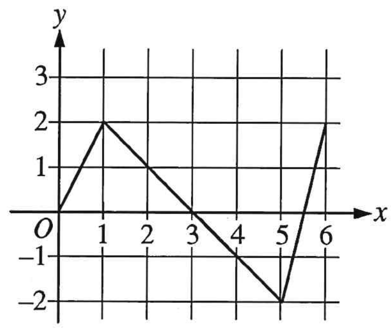
9. The function f has a first derivative given by $f'(x) = x(x-3)^2(x+1)$. At what values of x does f have a relative maximum?

(A) -1 only (B) 0 only (C) -1 and 0 only (D) -1 and 3 only (E) -1, 0, and 3

$$f(x) = \begin{cases} \dfrac{x^2 - 7x + 10}{b(x-2)} & \text{for } x \neq 2 \\ b & \text{for } x = 2 \end{cases}$$

10. Let f be the function defined above. For what value of b is f continuous at $x = 2$?

(A) -3 (B) $\sqrt{2}$ (C) 3 (D) 5 (E) There is no such value of b.



Graph of f'

11. For $0 \le x \le 6$, the graph of f', the derivative of f, is piecewise linear as shown above. If $f(0) = 1$, what is the maximum value of f on the interval?

(A) 1 (B) 1.5 (C) 2 (D) 4 (E) 6

---

12. Let f be the function given by $f(x) = 9^x$. If four subintervals of equal length are used, what is the value of the right Riemann sum approximation for $\int_0^2 f(x)\,dx$?

(A) 20 (B) 40 (C) 60 (D) 80 (E) 120

13. $\dfrac{d}{dx}\left(\dfrac{x+1}{x^2+1}\right) =$

(A) $\dfrac{x^2+2x-1}{\left(x^2+1\right)^2}$

(B) $\dfrac{-x^2-2x+1}{x^2+1}$

(C) $\dfrac{-x^2-2x+1}{\left(x^2+1\right)^2}$

(D) $\dfrac{3x^2+2x+1}{\left(x^2+1\right)^2}$

(E) $\dfrac{1}{2x}$

14. The velocity of a particle moving along the x-axis is given by $v(t) = \sin(2t)$ at time t. If the particle is at $x = 4$ when $t = 0$, what is the position of the particle when $t = \frac{\pi}{2}$?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

15. The function $y = g(x)$ is differentiable and increasing for all real numbers. On what intervals is the function $y = g(x^3 - 6x^2)$ increasing?

(A) $(-\infty, 0]$ and $[4, \infty)$ only

(B) $[0, 4]$ only

(C) $[2, \infty)$ only

(D) $[6, \infty)$ only

(E) $(-\infty, \infty)$

---

16. $\lim_{x \to 3^-} \dfrac{|x-3|}{x-3}$ is

(A) -3 (B) -1 (C) 1 (D) 3 (E) nonexistent

17. If $f(x) = ae^{-ax}$ for $a > 0$, then $f'(x) =$

(A) e^{-ax}

(B) ae^{-ax}

(C) a^2e^{-ax}

(D) $-ae^{-ax}$

(E) $-a^2e^{-ax}$

GO ON TO THE NEXT PAGE.

18. A student attempted to solve the differential equation $\frac{dy}{dx} = xy$ with initial condition $y = 2$ when $x = 0$. In which step, if any, does an error first appear?

Step 1: $\int \frac{1}{y}\,dy = \int x\,dx$

Step 2: $\ln|y| = \frac{x^2}{2} + C$

Step 3: $|y| = e^{x^2/2} + C$

Step 4: Since $y = 2$ when $x = 0$, $2 = e^0 + C$.

Step 5: $y = e^{x^2/2} + 1$

(A) Step 2

(B) Step 3

(C) Step 4

(D) Step 5

(E) There is no error in the solution.

19. For what values of x does the graph of $y = 3x^5 + 10x^4$ have a point of inflection?

(A) $x = -\frac{8}{3}$ only

(B) $x = -2$ only

(C) $x = 0$ only

(D) $x = 0$ and $x = -\frac{8}{3}$

(E) $x = 0$ and $x = -2$

---

20. $\lim_{x \to 2} \frac{\ln(x+3) - \ln(5)}{x - 2}$ is

(A) 0 (B) $\frac{1}{5}$ (C) $\frac{1}{2}$ (D) 1 (E) nonexistent

GO ON TO THE NEXT PAGE.

21. Functions w, x, and y are differentiable with respect to time and are related by the equation $w = x^2y$. If x is decreasing at a constant rate of 1 unit per minute and y is increasing at a constant rate of 4 units per minute, at what rate is w changing with respect to time when $x = 6$ and $y = 20$?

(A) -384 (B) -240 (C) -96 (D) 276 (E) 384

---

22. Let f be the function defined by $f(x) = 2x^3 - 3x^2 - 12x + 18$. On which of the following intervals is the graph of f both decreasing and concave up?

(A) $(-\infty, -1)$ (B) $\left(-1, \frac{1}{2}\right)$ (C) $(-1, 2)$ (D) $\left(\frac{1}{2}, 2\right)$ (E) $(2, \infty)$

$$f(x) = \begin{cases} 3x + 5 & \text{when } x < -1 \\ -x^2 + 3 & \text{when } x \geq -1 \end{cases}$$

23. If f is the function defined above, then $f'(-1)$ is

(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

---

24. Let f be the function defined by $f(x) = \dfrac{(3x+8)(5-4x)}{(2x+1)^2}$. Which of the following is a horizontal asymptote to the graph of f?

(A) $y = -6$

(B) $y = -3$

(C) $y = -\dfrac{1}{2}$

(D) $y = 0$

(E) $y = \dfrac{3}{2}$

25. If $y = x^2 - 2x$ and $u = 2x + 1$, then $\dfrac{dy}{du} =$

(A) $\dfrac{2\left(x^2 + x - 1\right)}{(2x + 1)^2}$ (B) $6x^2 - 3x - 2$ (C) $4x$ (D) $x - 1$ (E) $\dfrac{1}{x - 1}$

---

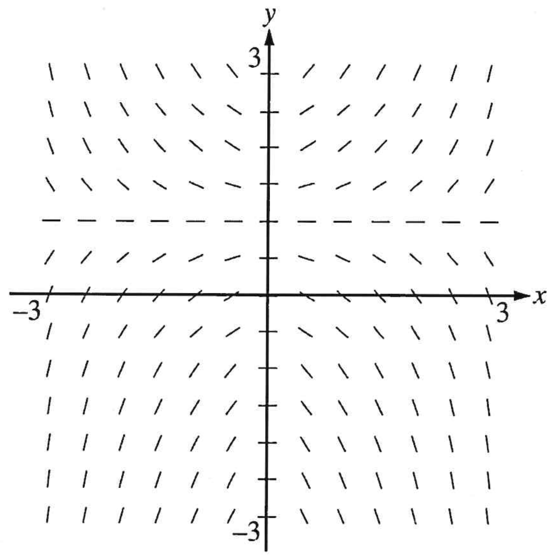
26. For $x > 0$, $\dfrac{d}{dx}\displaystyle\int_1^{\sqrt{x}} \frac{1}{1 + t^2}\,dt =$

(A) $\dfrac{1}{2\sqrt{x}\,(1 + x)}$ (B) $\dfrac{1}{2\sqrt{x}\left(1 + \sqrt{x}\right)}$ (C) $\dfrac{1}{1 + x}$ (D) $\dfrac{\sqrt{x}}{1 + x}$ (E) $\dfrac{1}{1 + \sqrt{x}}$

GO ON TO THE NEXT PAGE.

27. A particle moves on the x-axis so that at any time t, $0 \leq t \leq 1$, its position is given by $x(t) = \sin(2\pi t) + 2\pi t$. For what value of t is the particle at rest?

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 1



28. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy - x$

(B) $\frac{dy}{dx} = xy + x$

(C) $\frac{dy}{dx} = y - x^2$

(D) $\frac{dy}{dx} = (y - 1)x^2$

(E) $\frac{dy}{dx} = (y - 1)^3$

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.